IPPT Reports on Fundamental Technological Research, **47**, 4, 25–29, 2013
Institute of Fundamental Technological Research, Warszawa
Polish Academy of Sciences

ADVANCES IN DEVELOPMENT OF DEDICATED EVOLUTIONARY ALGORITHMS FOR LARGE NON-LINEAR CONSTRAINED OPTIMIZATION PROBLEMS

Maciej GŁOWACKI[1,2], Janusz ORKISZ[2]

[1] **Faculty of Physics Astronomy and Applied Computer Science**
Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland
e-mail: mglowac@gmail.com

[2] **Institute for Computational Civil Engineering**
Cracow University of Technology
Warszawska 24, 31-155 Kraków, Poland

Received 12 November 2013, in final form 27 November 2013
Published online 28 November 2013

Abstract: Efficient optimization algorithms are of great importance in many scientific and engineering applications. This paper considers development of dedicated Evolutionary Algorithms (EA) based approach for solving large, non-linear, constrained optimization problems. The EA are precisely understood here as decimal-coded Genetic Algorithms consisting of three basic operators: selection, crossover and mutation, followed by several newly developed calculation speed-up techniques. Efficiency increase of the EA computations may be obtained in several ways, including simple concepts proposed here like: solution smoothing and balancing, a posteriori solution error analysis, non-standard use of distributed and parallel calculations, and step-by-step mesh refinement. Efficiency of the proposed techniques has been evaluated using several benchmark tests. These preliminary tests indicate significant speed-up of the large optimization processes involved. Considered are applications of the EA to the sample problem of residual stresses analysis in elastic-plastic bodies being under cyclic loadings, and to a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental data.

Key words: Evolutionary Algorithms, large non-linear constrained optimization, solution efficiency increase.

1. Introduction

In this paper we consider development of computational efficiency of the optimization approach based on a modified Evolutionary Algorithm (EA), with several acceleration concepts introduced in it. Our long-term research [1–3] is oriented towards development of efficient tool for numerical solution of a wide class of large, non-linear, constrained optimization problems.

Optimization problems may be solved by means of either deterministic or probabilistic methods. In contrast to the deterministic methods, the EA may be successfully applied with similar efficiency to both the convex and non-convex problems. However, general efficiency of the standard EA is rather low. Therefore, the main objective of our research is to develop means of an essential acceleration of the EA based solution approach for large, non-linear, constrained optimization problems. Moreover, the improved EA should provide possibility of solving such optimization problems, when the standard EA methods fail. In order to obtain a significant efficiency increase of the standard EA, several new acceleration techniques were introduced in it [1–3]. Chosen standard acceleration techniques are considered as well [4–7]. Our research involves analysis of several benchmark problems that allow for a wide investigation of various acceleration concepts. Finally, we take into account the following long-term applications in our research: residual stress analysis in railroad rails and vehicle wheels [8–10], as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental data [10, 11].

2. Problem formulation

We consider large, non-linear, constrained optimization problems, where a searched function is given in the discrete form, e.g., expressed in terms of its nodal values. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. In order to achieve an acceleration of the optimization process, the modified EA will use smoothing and balancing techniques, solution averaging, a posteriori error analysis, non-standard parallel and distributed calculations, as well as adaptively refined series of meshes, and possible combinations of the above.

3. Dedicated evolutionary algorithms for large optimization problems

The EA is precisely understood here as a decimal-coded Genetic Algorithm consisting of three basic operators: selection, crossover and mutation [12]. Significant acceleration of the standard EA may be achieved in various ways including, e.g., hybrid algorithms [4, 5] combining the EA with deterministic methods, development of new, problem-oriented operators as well as standard distribution and parallelization of computations [6, 7]. Moreover, we have recently proposed, and preliminarily tested, several acceleration techniques based on simple concepts [1–3]. Particular attention has been paid to the application of a posteriori solution error estimation and related techniques [1]. We are presenting here a brief overview of the proposed solution approach, including advances in its development, and numerical results of a sample benchmark problem.

3.1. Smoothing and balancing. Available additional information about solution smoothness may be used in various ways [2]. For instance, an extra procedure based on the Moving Weighted Least Squares (MWLS) technique [13, 14] or any other equivalent approximation method is applied in order to smooth the raw results obtained from the standard EA procedure. In problems of mechanics each smoothing may result in the global equilibrium loss of a considered body. The equilibrium may be restored by means of an artificial balancing of body forces performed after the smoothing [2, 3].

3.2. A posteriori error analysis. Proposed a posteriori error analysis [1, 15] is based on a stochastic nature of evolutionary computations. Reference solutions required to estimate local errors are obtained by weighted averaging of the best solutions taken from independent EA processes. Information about the magnitude and distribution of the local errors is used to improve crossover and mutation operators intensifying calculations in large error zones. Information about the global errors may be also introduced as an additional selection criterion. Such a posteriori error analysis is well supported by non-standard parallel and distributed calculations.

3.3. Step by step mesh refinement. When using an adaptive step-by-step mesh refinement, the analysis starts from a coarse mesh, allowing to obtain a solution much faster than in the fully dense grid case. When decreasing the number of nodes, the time spent on each iteration is reduced. However, such solution may usually be not precise enough. In order to increase its precision, the mesh is refined by inserting new nodes, based on the results of the error analysis. The initial function values for these nodes are found by using an approximation built upon the coarse mesh nodal values. A general approach for most optimization problems may be obtained, e.g., by using the MWLS approximation [13, 14]. Furthermore, the mesh refinement may also be used for generation of reference solutions in the a posteriori error analysis. Such a combined strategy, using both techniques mentioned above, involves the following steps:

1. Evaluation of the solution on a coarse mesh.
2. Smoothing of this rough solution.
3. Mesh refinement and approximation of the initial values in inserted nodes.
4. Use of the obtained solution as an initial reference for the error estimation.
5. Use of the weighted solution averaging for further reference solution generation, and the a posteriori error analysis.
6. Repetition of the above procedure until sufficiently dense mesh is reached.

4. Numerical results of benchmark problems

A variety of benchmark problems was chosen in order to evaluate the efficiency of the proposed acceleration techniques. In particular, we have analyzed the residual stresses in an elastic-perfectly plastic bar subject to cyclic bending, and in the thick-walled cylinder made of the same material but subject to a cyclic pressure. Both problems were analyzed as 1D (taking into account existing symmetries), and as 2D ones as well. For instance, the following optimization problem given in the polar coordinates for residual stresses in a thick-walled cylinder was analyzed [9].

Find the minimum of the total complementary energy:

$$\min_{\sigma_r^r,\sigma_t^r,\sigma_z^r} \frac{1}{2E} 2\pi L \int_a^b \left[(\sigma_r^r - \sigma_t^r)^2 + (\sigma_t^r - \sigma_z^r)^2 + (\sigma_z^r - \sigma_r^r)^2 \right] r dr, \tag{1}$$

subject to the equilibrium equation:

$$\frac{\partial \sigma_r^r}{\partial r} + \frac{\sigma_r^r - \sigma_t^r}{r} = 0, \tag{2}$$

boundary conditions:

$$\sigma^r_{r|a} = 0, \qquad \sigma^r_{r|b} = 0, \tag{3}$$

the incompressibility equation:

$$\sigma^r_z = \nu(\sigma^r_r + \sigma^r_t), \tag{4}$$

the yield condition:

$$\phi\left(\sigma^r_r, \sigma^r_t, \sigma^r_z, \sigma^E\right) \leq \sigma_Y, \tag{5}$$

where σ^r_r, σ^r_t, σ^r_z are respectively the radial, circumferential and longitudinal residual stresses, $\sigma^E = \{\sigma^E_r, \sigma^E_t, \sigma^E_z\}$ is the purely elastic solution of the problem, σ_Y is the yield stress, a, b are respectively the internal and external cylinder radii, L is its length, and E is the Young modulus.

Typical results are shown in Fig. 1, which presents results of residual stresses obtained from the thick-walled cylinder 1D analysis. They were obtained by the accelerated EA recently developed by us, using a series of meshes and the a posteriori error analysis. The process started with seven nodes (Fig. 1), and was continued until the number of 3073 nodes was reached. When using the standard EA, even for much smaller number of nodes, the solution cannot be obtained in a reasonable number of iterations. In comparison to the standard EA procedure, the same level of the mean solution error was obtained about 120 times faster. Furthermore, the precision of the final solution has increased about 90 times.

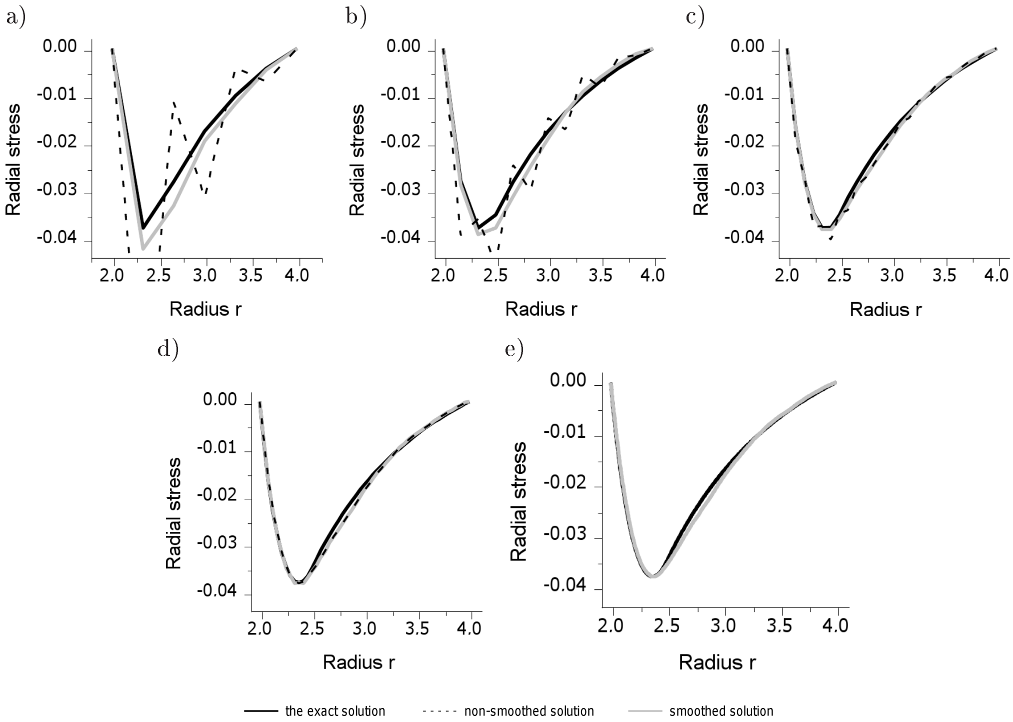

Fig. 1. Solutions in the cross-section obtained for subsequent initial meshes with 7, 13, 25, 49 nodes respectively (a–d), and final 3073 nodes (e).

5. Final remarks

The general objective of this research is development of the EA method for efficient solution of large, non-linear, constrained optimization problems. Preliminary results of the executed tests clearly show a significant improvement of the optimization process in comparison with the standard EA. It is also worth noticing, that the improved EA allowed obtaining solutions in cases when the standard EA failed, e.g., for large number of decision variables.

Future research includes continuation of various efforts oriented towards an improvement of the EA-based optimization approach, analysis of further benchmark problems, and application of such developed method to residual stresses analysis in railroad rails and vehicle wheels [9, 10]. The physically based smoothing of experimental data is also expected.

References

1. J. Orkisz, M. Głowacki, On acceleration of evolutionary algorithms taking advantage from aposteriori error analysis, *Computing and Informatics*, accepted for publication.
2. J. Orkisz, M. Głowacki, On certain improvements for evolutionary algorithms applied to residual stresses analysis, in *CMM 2013 Short Papers*, T. Łodygowski et al. [Eds.], Poznań University of Technology, Poznań, MS05-7-8, 2013.
3. J. Orkisz, M. Głowacki, On stress reconstruction using experimental data the PBA and accelerated EA, in *ECCOMAS 2013. Conference Proceedings*, Z. Waszczyszyn, L. Ziemiański [Eds.], Rzeszow University of Technology, Rzeszów, 43–44, 2013.
4. T. Burczynski, P. Orantek, Evolutionary and hybrid algorithms, in *Neural Networks, Genetic Algorithms, Fuzzy Sets* [in Polish], BEL, Rzeszow, 99–117, 1999.
5. C. Grosan, A. Abraham, H. Ishibuchi [Eds.], Hybrid evolutionary algorithms, *Studies in Computational Intelligence* **75**, Springer, 2007.
6. W. Kus, T. Burczynski, Parallel bioinspired algorithms in optimization of structures, *Lecture Notes in Computational Sciences* **4967/2008**, Springer, 1285–1292, 2008.
7. N. Nedjah, E. Alba, L. Mourelle [Eds.], Parallel evolutionary computations, *Studies in Computational Intelligence* **22**, Springer, 2006.
8. R. Hill, *The mathematical theory of plasticity*, Oxford University Press, New York, 2004.
9. J. Orkisz, Prediction of actual residual stresses by constrained minimization of energy, in *Residual Stress in Rails*, Orringer O., Orkisz J., Swiderski Z. [Eds.], Kluwer Academic Publisher **2**, 101–124, 1992.
10. J. Orkisz *et al.*, *Development of advanced methods for theoretical prediction of shakedown stress states and physically based enhancement of experimental data*, US DOT report, DTFR53-03-G-00012, Cracow, 2004.
11. W. Karmowski, J. Orkisz, Physically Based Method of Enhancement of Experimental Data – Concepts, Formulation and Application to Identification of Residual Stresses, Proc. of IUTAM Symp. on Inverse Problems in Engng Mech., Tokyo, 1992, *On Inverse Problems in Engineering Mechanics*, M. Tanaka, H.D. Bui [Eds.], Springer-Verlag, pp. 61–70, 1993.
12. A.P. Engelbrecht, *Computational intelligence: an introduction*, Wiley, Chichester, 2007.
13. J. Orkisz, T. Liszka, The finite difference method at arbitrary irregular grids and itsapplications in applied mechanics, *Computers and Structures* **11**, 83–95, 1980.
14. K. Salkauskas, P. Lancaster, *Curve and surface fitting*, Academic Press, 1990.
15. M. Ainsworth, J.T. Oden, Aposteriori error estimation in finite element analysis, *Computer Methods in Applied Mechanics and Engineering* **142**, 1–88, 1997.